Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed.
Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

(d) The amount of unprocessed gravel at time t is given by
$A(t) = 500 + \int_0^t (G(s) - 100)\,ds.$

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G(t) = 90 + 45\cos\frac{t^2}{18}$

$G'(t) = -5t\sin\left(\frac{t^2}{18}\right)$

$G'(5) = -24.588$ tons/hr ^{2}

This means that the rate at which unprocessed gravel arrives at the processing plant is changing by -24.588 tons per hour per hour, or decreasing by 24.588 tons per hour per hour, at t = 5 hours.

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(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 \left[90 + 45\cos\left(\frac{t^2}{18}\right)\right] dt = 825.551$ tons

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(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

Let $V(t)$ be the amount of unprocessed gravel.

$V'(t)$ is the rate at which the amount of unprocessed gravel is changing.

$$V'(t) = G(t) - 100$$

$$V'(5) = G(5) - 100$$

$$= 90 + 45\cos\left(\frac{5^2}{18}\right) - 100$$

$$V'(5) = -1.859$$

Since $V'(5)$ is negative, the amount of unprocessed gravel is decreasing at time $t = 5$ hours

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$V'(t) = 0$ at $t = ?$

$$0 = G(t) - 100$$

$$100 = 90 + 45\cos\left(\frac{t^2}{18}\right)$$

$$10 = 45\cos\left(\frac{t^2}{18}\right)$$

$$t = 4.923$$

$$V(t) - V(0) = \int_0^t (G(x) - 100)\,dx$$

$$V(t) = \int_0^t (G(x) - 100)\,dx + V(0)$$

$V(0) = 500$

$V(4.923) = 635.376$

$V(8) = 525.551$

Since $V(t)$ is on a closed interval $[0, 8]$, the maximum amount must occur at an endpoint or at a critical value. After evaluating the amount of unprocessed gravel at $t = 0$, $t = 4.923$, and $t = 8$, the amount of unprocessed gravel is highest at $t = 4.923$, with 635.376 tons of unprocessed gravel.

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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using ~~correct units,~~ interpret your answer in ~~the context~~ of the problem.

$G'(5) = -24.588$ tons/hour²

$G'(5)$ represents the rate of change in tons/hour² of the rate at which unprocessed gravel arrives

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 90+45\cos\left(\frac{t^2}{18}\right)dt$

$= 825.551$ tons

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(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

rate gravel is arriving $= G(5) = 98.141$ tons/hour

rate gravel is being processed $= 100$ tons/hour

The amount of unprocessed gravel at the plant at time $t=5$ hours is decreasing since the rate at which the gravel is being processed is greater than the rate at which it is arriving.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$500+\int_0^t 90+45\cos\left(\frac{t^2}{18}\right)$

$90+45\cos\left(\frac{t^2}{18}\right)=100$

$t=4.923$

$500+\int_0^{4.923} 90+45\cos\left(\frac{t^2}{18}\right)dt$

1127.676 tons

this occurs ~~at the time~~ when the rate of the amount arriving equals the rate it is being processed

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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$$G'(t) = -5t\sin\left(\frac{t^2}{18}\right)$$

$$G'(5) = -5(5)\sin\left(\frac{25}{18}\right)$$

$$G'(5) = -25\sin\left(\frac{25}{18}\right) \approx \boxed{-24.588 \text{ tons per hour}^2}$$

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$$\int_0^8 \left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)dt = \boxed{825.551 \text{ tons}}$$

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(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

$G(5) = 90 + 45\cos\left(\frac{5^2}{18}\right)$

$G(5) = 98.1408$

positive amount arriving

increasing

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

at $t=8$, max amount

~~$G(8) = 90 + 45\cos\left(\frac{8^2}{18}\right)$~~

at end of day

$\int 90 + 45\cos\left(\frac{t^2}{18}\right)dt = 825.551$ tons

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Question 1

Overview

This problem provided information related to the amount of gravel at a gravel processing plant during an eight-hour period. The function G, given by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, models the rate, in tons per hour, at which gravel arrives at the plant. The problem also stated that gravel is processed at a constant rate of 100 tons per hour. In part (a) students were asked to find $G'(5)$, the derivative of G at time $t = 5$. This value is negative, so students should have interpreted the absolute value of this number as the rate at which the rate of arrival of gravel at the plant is decreasing, in tons per hour per hour, at time $t = 5$. In part (b) students were asked to find the total amount of unprocessed gravel arriving at the plant over the eight-hour workday. Students should have evaluated the definite integral $\int_0^8 G(x)\,dx$, recognizing that integrating the rate at which gravel arrives over a time interval gives the net amount of gravel that arrived over that time interval. Part (c) asked whether the amount of unprocessed gravel at the plant is increasing or decreasing at time $t = 5$. Students determined whether the rate at which unprocessed gravel is arriving is greater than the rate at which gravel is being processed, i.e., whether $G(5) > 100$. Part (d) asked students to determine the maximum amount of unprocessed gravel at the plant during this workday. Because the amount of unprocessed gravel at the plant at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$, students needed to identify the critical points of this function (where $G(t) = 100$) and to determine the global maximum on the interval $[0, 8]$. This could have been done by observing that there is a unique critical point on the interval, which is a maximum, and determining the amount of unprocessed gravel at the plant at that time, or by computing the amount of unprocessed gravel at this critical point and at the endpoints for comparison.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student presents a correct value for $G'(5)$ and earned the first point. The student does not address time $t = 5$ in the interpretation of the value, so the second point was not earned. In parts (b) and (c), the student's work is correct. In part (d) the first point was earned for considering where $G(t) = 100$. The student does not correctly determine the maximum amount of gravel, so the second point was not earned. A justification for a global maximum was not provided, so the third point was not earned.

Sample: 1C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (b). In part (a) the student presents a correct value for $G'(5)$ and earned the first point. The student does not provide an interpretation of this value, so the second point was not earned. In part (b) the student's work is correct. In part (c) the student ignores the rate at

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Question 1 (continued)

which gravel was being processed and did not earn either point. In part (d) the student again does not consider the rate at which gravel was being processed. The student did not earn any points in this part.

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Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $\lvert v(t) \rvert = 2$ on $2 \le t \le 4$. $t = 3.128$ (or 3.127) and $t = 3.473$	$2 : \begin{cases} 1 : \text{considers } \lvert v(t) \rvert = 2 \\ 1 : \text{answer} \end{cases}$
(b) $s(t) = 10 + \int_0^t v(x)\,dx$ $s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$	$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$
(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$ $v(t)$ changes sign from negative to positive at time $t = 0.536033$. $v(t)$ changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$
(d) $v(4) = -11.475758 < 0,\ \ a(4) = v'(4) = -22.295714 < 0$ The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.	2 : conclusion with reason

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2A₁

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$|v(t)| = 2 \quad 2 \le t \le 4$

$t = 3.128, 3.473$

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

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(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

$v(t) = 0$

$t = 0.536, 3.318$

the particle changes direction at 0.536 because $v(t)<0$ for $(0, 0.536)$ and $v(t)>0$ for $(0.536, 3.318)$. The particle changes direction at 3.318 because $v(t)>0$ for $(0.536, 3.318)$ and $v(t)<0$ for $t>3.318$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(4)<0$

$a(4)=v'(4)<0$

The speed is increasing at $t=4$ because both $v(4)$ and $a(4)=v'(4)$ are negative.

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$2 = -2 + (t^2 + 3t)^{6/5} - t^3$

$0 = -4 + (t^2 + 3t)^{6/5} - t^3$

$t = 3.128$

$\boxed{t = 3.128}$

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$\boxed{s(t) = 10 + \int_0^t v(x)\,dx}$

$s(5) = 10 + \int_0^5 v(x)\,dx$

$\boxed{s(5) = -9.207}$

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2B₂

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

The particle changes direction when velocity changes sign. This occurs at time $t = 3.318$ as the velocity changes from positive to negative.

$0 = -2 + (t^2+3t)^{1.2} - t^3$

$\boxed{t = 3.318}$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(t) = -2 + (t^2+3t)^{1.2} - t^3$

$a(t) = 1.2\sqrt[5]{t^2+3t}\,(2t+3) - 3t^2$

$v(4) = -2 + (16+12)^{1.2} - 64$

$v(4) = -11.476$

$a(4) = 1.2\sqrt[5]{16+12}\,(8+3) - 48$

$a(4) = -22.296$

The speed of a particle is increasing if acceleration and velocity have the same sign and decreasing if acceleration and velocity have different signs. The speed of the particle is increasing at time $t=4$ because, at that point, both the velocity and the acceleration are negative.

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 2C1

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

x y

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$2 = -2 + (t^2 + 3t)^{6/5}$

$0 = (t^2 + 3t)^{6/5} - t^3 - 4$

$\rightarrow$ graph

at $t = 3.1276299$

$\boxed{t = 3.128}$

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(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = \int v(t) \rightarrow -t^2 + \frac{5}{11}\left(\frac{t^3}{3} + \frac{3t^2}{2}\right)^{11/5} - \frac{t^4}{4} + C$

$10 = 0 + \frac{5}{11}(0+0)^{11/5} - 0 + C \rightarrow C = 10$

@ $s(5) = -25 + 6829.210659 - 156.25 + 10$

$\boxed{s(5) = 6657.961}$

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2C 2

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

The particle changes direction when the velocity changes sign

$0 = -2 + (t^2+3t)^{6/5} - t^3 \rightarrow$ graph

$v(0) =$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(4) = -2 + (4^2+3(4))^{6/5} - 4^3$

$v(4) = -11.476$

$a(t) = \frac{6}{5}(t^2+3t)^{1/5} \cdot (2t+3) - 3t^2$

$a(4) = -22.296$

The speed is increasing because acceleration and velocity are both negative.

~~$S(4) = 1975.737$~~

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Question 2

Overview

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 5$. The particle's position at time $t = 0$ is given, and the velocity function v is provided. Part (a) asked students to determine the times when the speed of the particle is 2, which required determining where the velocity function is ± 2 or where the absolute value of the velocity function is 2. In part (b) students were asked to provide an integral expression for the position $s(t)$ and then to use this expression to find the position of the particle at time $t = 5$. Students should have recognized that the position is given by $s(t) = s(0) + \int_0^t v(x)\,dx$ and then evaluated $s(5)$ to determine the position at time $t = 5$. Part (c) asked students to determine all times t, $0 \le t \le 5$, at which the particle changes direction. Students needed to determine where $v(t)$ changes sign. In part (d) students were asked whether the speed of the particle is increasing or decreasing at time $t = 4$. Students should have evaluated both the velocity and the acceleration functions at time $t = 4$. Because $v(4) < 0$ and $a(4) < 0$, the speed of the particle is increasing.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is incorrect. In part (b) the student's work is correct. In part (c) the student earned the point for considering $v(t) = 0$ and 1 point for a single correct answer with correct justification. In part (d) the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: 1 point in part (c) and 2 points in part (d). In parts (a) and (b), the student's work is incorrect and did not earn any points. In part (c) the student earned the point for considering $v(t) = 0$. In part (d) the student's work is correct.

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Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval)

$\dfrac{C(4) - C(2)}{4 - 2} = \dfrac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\dfrac{1}{6}\int_0^6 C(t)\,dt \approx \dfrac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$

$= \dfrac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$

$= \dfrac{1}{6}(60.6) = 10.1$ ounces

$\dfrac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

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3A1 3A1

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$C'(3.5) \approx \frac{C(4)-C(3)}{4-3} = \frac{12.8-11.2}{1} = 1.6 \ \frac{oz}{min}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

since $C(t)$ is differentiable for all values in $[2,4]$, we can say that there must be some value of t in $(2,4)$, such as $t=a$, such that $C'(a) = \frac{C(4)-C(2)}{4-2} = \frac{12.8-8.8}{2} = 2$ by the mean value theorem

So, yes!

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3A2 3A2

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}\cdot 2\cdot(5.3+11.2+13.8)$$

$$\approx 10.1 \text{ oz}$$

this is the average value, in oz, of the amount of coffee in the cup over the interval $0 \le t \le 6$

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$$B'(t) = \frac{32}{5}e^{-0.4t}$$

$$B'(5) = \frac{32}{5}e^{-2}$$

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3B₁ 3B₁

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t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$\frac{f(4)+f(3)}{2} = \frac{12.8+11.2}{2} = \boxed{12\text{ oz}}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

Yes, according to the Mean Value Theorem

$$c'(t) = \frac{f(b)-f(a)}{b-a}$$

$$\boxed{c'(t) = \frac{12.8-8.8}{4-2} = \frac{4}{2} = 2}$$

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3B2 3B2

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

stepsize of 2

$(5.3)2 + (11.2)2 + (13.8)2$

$10.6 + 22.4 + 27.6 = 60.6$

$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}(60.6) = 10.1\text{ oz}$

Over the course of six hours, the coffeemaker's average production is 10.1 oz per hour.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$\frac{16}{.4}$

$B'(t) = 6.4e^{-0.4t}$

$\boxed{B'(5) = 6.4/e^2}$

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t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$C'(3.5) = \frac{12.8-11.2}{4-3} = \frac{1.6}{1} = \boxed{1.6 = C'(3.5)}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

$$C'(c) = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{14.5-8.8}{6-2} = \frac{5.7}{4} < 2$$

$$\begin{array}{r} 14.7 \\ -9.0 \\ \hline \end{array}$$

No because $\frac{5.7}{4}$ is less than 2

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(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$\Delta x \cdot h + \Delta x \cdot h + \Delta x \cdot h$

$2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8$

the meaning of $\frac{1}{6}\int^6 C(t)dt$ is the amount of ounces of water that was poured into the cup over the course of 0 seconds to 6 seconds.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$\frac{2}{5} = .4$

$B(t) = 16 - 16e^{-.4t}$

$B'(t) = -16 \cdot -.4e^{-.4t}$

$B'(t) = \frac{+32}{5}e^{-.4(t)}$

$-\frac{2}{5} \cdot 5$

$B'(5) = \frac{32}{5}e^{-.4(5)}$

$B'(5) = \frac{32}{5}e^{-1}$

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Question 3

Overview

In this problem, a table was provided giving values of a differentiable function C at selected times between $t = 0$ and $t = 6$ minutes, where $C(t)$ represented the amount of coffee, in ounces, in a cup at time t. Part (a) asked students to approximate the derivative of the function C at $t = 3.5$ and to indicate units of measure. Because $t = 3.5$ fell between values of t given in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that is supported by the table. Students should have recognized this derivative as the rate at which the amount of coffee in the cup is increasing, in ounces per minute, at time $t = 3.5$. Part (b) asked students whether there is a time t, $2 \le t \le 4$, at which $C'(t) = 2$. Students should have recognized that the hypotheses for the Mean Value Theorem hold because C is differentiable and then applied the theorem to the function on the interval $[2, 4]$ to conclude that there must be such a time t. Part (c) asked for an interpretation of $\frac{1}{6}\int_0^6 C(t)\,dt$ and a numeric approximation to this expression using a midpoint sum with three subintervals of equal length as indicated by the data in the table. Students should have recognized this expression as providing the average amount, in ounces, of coffee in the cup over the 6-minute time period. Students needed to use the values in the table at times $t = 1$, $t = 3$, and $t = 5$, with interval length 2, to compute this value. In part (d) students were given a symbolic expression for a function B that modeled the amount of coffee in the cup on the interval $0 \le t \le 6$. Students were asked to use this model to determine the rate at which the amount of coffee in the cup is changing when time $t = 5$. This was answered by computing the value $B'(5)$.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student does not construct a numeric difference quotient, so no points were earned. The use of f instead of C is ignored. In part (b) the student's work is correct. The use of f instead of C is ignored because the student correctly uses C-values from the table and invokes the Mean Value Theorem. In part (c) the student earned the first 2 points. The interpretation point was not earned because the student does not commit to ounces as the units. The student also uses "ounces per hour" in the interpretation. In part (d) the student's work is correct.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (c), and 1 point in part (d). In part (a) the student earned the approximation point, but not the units point. In part (b) the student's work is incorrect. In part (c) the student earned the midpoint sum point, but not the approximation point. The interpretation does not include "average," so the third point was not earned. In part (d) the student has the correct derivative but incorrectly multiplies $-0.4(5)$, so the answer point was not earned.

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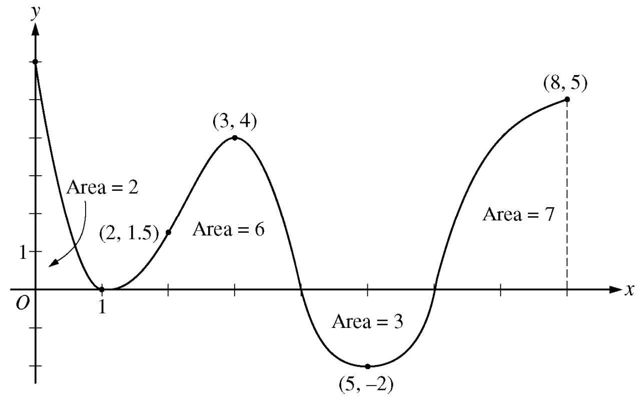
Question 4



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.	1 : answer with justification
(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint. $f(0) = f(8) + \int_8^0 f'(x)\,dx$ $= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$ $f(6) = f(8) + \int_8^6 f'(x)\,dx$ $= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$ $f(8) = 4$ The absolute minimum value of f on the closed interval $[0, 8]$ is -8.	$3 : \begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$
(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$ $g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$	$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$

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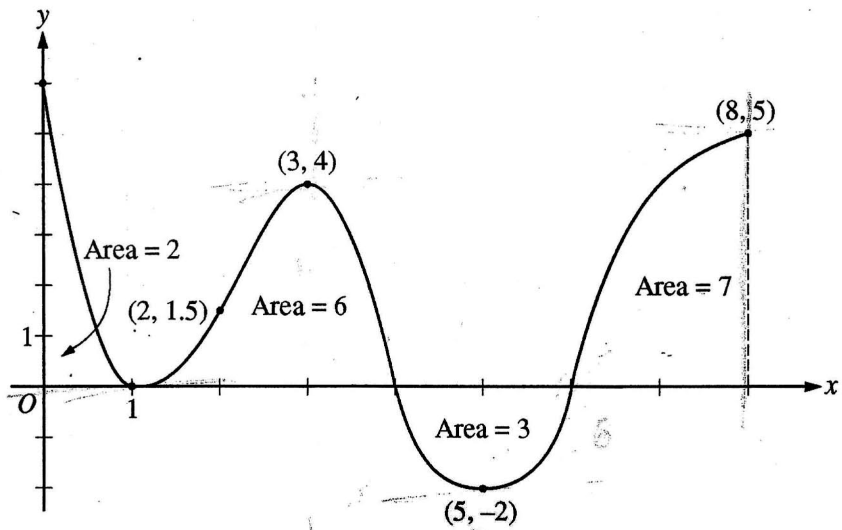

Graph of f'

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4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$x = 6$

f has a local minimumat $x = 6$, because the graph of f' changes from negative to positive at $x=6$, so using the first derivative test and ~~the fact the at x=6~~, f has a critical number, at $x=6$, f has a local minimum

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

local minimum = $x = 6$ $\qquad f(8) = 4$

$f(6) \rightarrow \int_6^8 f'(x)\,dx = 7 = f(8) - f(6) = 4 - f(6) \qquad f(6) = -3$

$f(0) \rightarrow \int_0^8 f'(x) = 12 = f(8) - f(0) = 4 - f(0) \qquad f(0) = -8$

The Absolute minimum value of f on the interval $0 \le x \le 8$ is -8 because it is the lowest value for f among the endpoints and critical numbers.

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

$\hookrightarrow f'' < 0$

The open intervals where the graph of f is both concave down and increasing is $(0,1) \cup (3,4)$, or $0<x<1$ and $3<x<4$, because using the graph of f', when the graph of f' is positive and the slope of f' is negative, that means that f is increasing and f'' is negative, so f is both concave down and increasing.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(f(3))^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot (4)$

$= 3\left(\frac{25}{\cancel{4}}\right)\cancel{4} = 75$

$g'(3) = 75$

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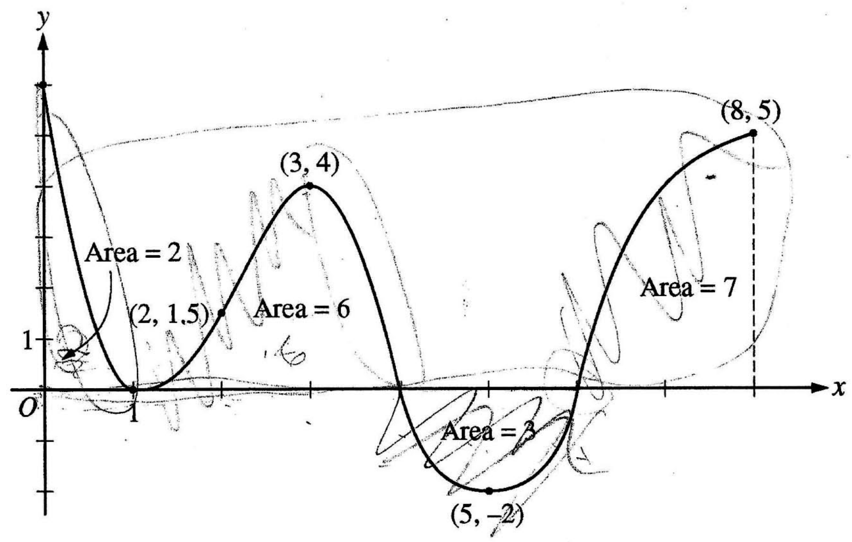

Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$x=6$ because the sign of f' changes from negative to positive.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

x	$\int_0^x f'(x)\,dx$
0	0
8	4
1	neither
4	relmax
6	5

$x=8$ because that is where $f(x)$ is the smallest.

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

f(x) is concave down and increasing when f'' is negative and f' is positive this occurs (0,1) and (3,4).

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$x_1 = 3$

$y_1 = -\frac{5}{2}$

$\boxed{m = -75}$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(-\frac{5}{2})^2 \cdot 4$

$g'(3) = -\frac{75}{\cancel{4}}\ \cancel{4}$

$g'(3) = \frac{-300}{4}$

$g'(3) = -75$

$\begin{array}{r} 75 \\ \times\ 4 \\ \hline 300 \end{array}$

$4\overline{)300}$ = 075

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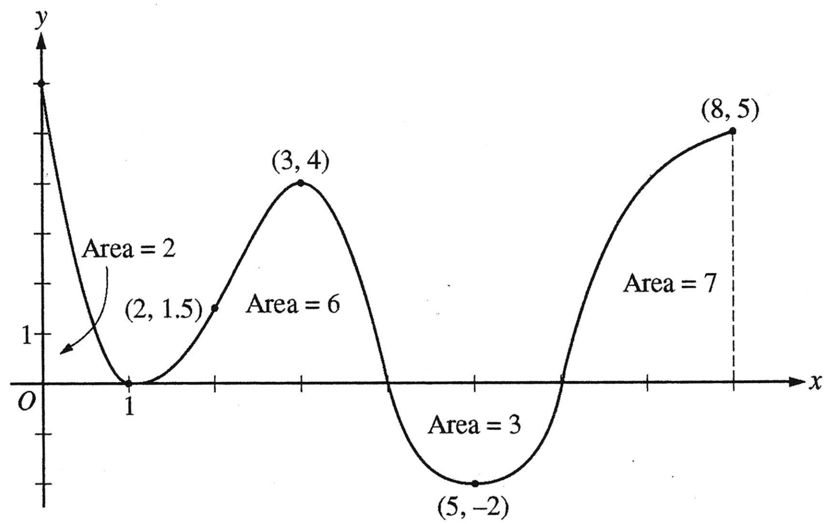

Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

Min of f is when f' changes from (-) to (+)

Min @ $x = 6$

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

x	$\int_0^8 f$
0	6
1	0
3	4
5	-2
8	5

$\boxed{\text{Min of } -2 \text{ at } x = 5}$

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

f is concave down when f'' is (-)

f is increasing when f' is (+)

Both from (0,1), u (3,4)

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(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$m = g'(x) = (f'(3))^3 = 4^3 = 48$

$x = 3$

$g(3) = (f(3))^3$

$\frac{-5^3}{3}$

$\frac{125}{9} = y$

$\begin{array}{r} 16 \\ 3 \\ \hline 48 \end{array}$

$\boxed{y - \frac{125}{9} = 48(x-3)}$

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Question 4

Overview

This problem described a function f that is defined and twice differentiable for all real numbers, and for which $f(8) = 4$. The graph of $y = f'(x)$ on $[0, 8]$ is given, along with information about locations of horizontal tangent lines for the graph of f' and the areas of the regions between the graph of f' and the x-axis over this interval. Part (a) asked for all values of x in the interval $(0, 8)$ at which f has a local minimum. Students needed to recognize that this occurs where f' changes sign from negative to positive. Part (b) asked for the absolute minimum value of f on the interval $[0, 8]$. Students needed to use the information about the areas provided with the graph, as well as $f(8)$, to evaluate $f(x)$ at 0 and at the local minimum found in part (a). Part (c) asked for the open intervals on which the graph of f is both concave down and increasing. Students needed to recognize that this is given by intervals where the graph of f' is both decreasing and positive. Students were to determine these intervals from the graph. Part (d) introduced a new function g defined by $g(x) = (f(x))^3$, and included that $f(3) = -\frac{5}{2}$. Students were asked to find the slope of the line tangent to the graph of g at $x = 3$. Students needed to recognize that this slope is given by $g'(3)$. In order to determine this value, students needed to apply the chain rule correctly and read the value of $f'(3)$ from the graph.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student earned the point for considering $x = 0$ and $x = 6$. The student does not report a correct answer and was not eligible for the justification point. In part (c) the student's work is correct. In part (d) the student earned the 2 points for $g'(x)$ but did not earn the answer point.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). In part (a) the student's work is correct. In part (b) the student does not consider $x = 0$ and $x = 6$, does not find the answer, and is not eligible for the justification point. In part (c) the student's work is correct. In part (d) the student makes a chain rule error in the derivative and did not earn the answer point.

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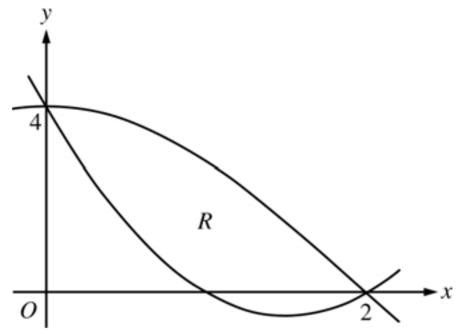
Question 5



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) $$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

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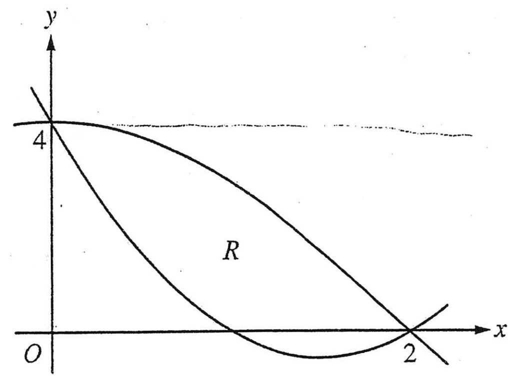

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$R = \int_0^2 \left(4\cos\left(\frac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right) dx$

$R = \left(\frac{16}{\pi}\sin\left(\frac{1}{4}\pi x\right) - \frac{2}{3}x^3 + 3x^2 - 4x\right)\Big|_0^2$

$R = \left(\frac{16}{\pi}\sin\left(\frac{1}{2}\pi\right) - \frac{16}{3} + 12 - 8\right) - \left(\frac{16}{\pi}\sin(0) - 0 + 0 - 0\right)$

$R = \frac{16}{\pi}(1) - \frac{16}{3} + 4$

$\boxed{R = \frac{16}{\pi} - \frac{4}{3}}$

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(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$V = \pi \int_0^2 \left((4 - (2x^2 - 6x + 4))^2 - (4 - 4\cos(\tfrac{1}{4}\pi x))^2 \right) dx$$

$$\boxed{V = \pi \int_0^2 \left((-2x^2 + 6x)^2 - (4 - 4\cos(\tfrac{1}{4}\pi x))^2 \right) dx}$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$V = \int_0^2 \left(4\cos(\tfrac{1}{4}\pi x) - (2x^2 - 6x + 4) \right)^2 dx$$

$$\boxed{V = \int_0^2 \left(4\cos(\tfrac{1}{4}\pi x) - 2x^2 + 6x - 4 \right)^2 dx}$$

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5B1

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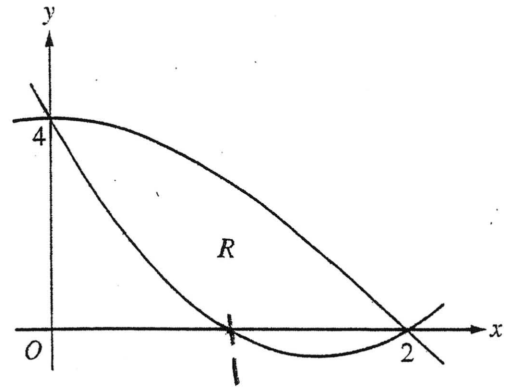

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$$\int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right) dx =$$

$$\int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - 2x^2 + 6x + 4\right) dx =$$

$$\pi \sin\left(\tfrac{1}{4}\pi x\right) - \tfrac{2}{3}x^3 + 3x^2 + 4x \Big|_0^2 =$$

$$\left(\pi\left(\sin\tfrac{\pi}{2}\right) - \tfrac{2}{3}(2)^3 + 3(2)^2 + 4(2)\right) =$$

$$\pi - \frac{16}{3} + 12 + 8 =$$

$$\boxed{\pi + \frac{44}{3} = R}$$

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5B2

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(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

V =

$$V = \pi \int_0^4 \left(\left[4 - (2x^2 - 6x + 4)\right]^2 - \left[4 - (4\cos(\tfrac{1}{4}\pi x))\right]^2 \right) dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2 - 6x + 4) \right)^2 \right) dx$$

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5C,

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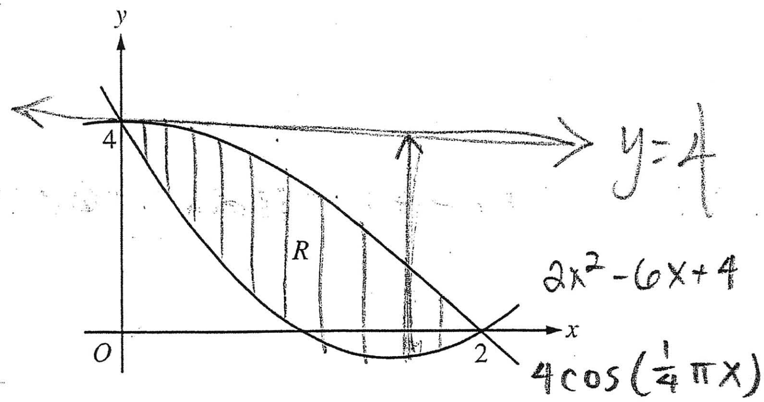

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$\frac{1}{4\pi}(1) + x(0)$

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right)\right) - \left(2x^2 - 6x + 4\right)\right) dx$$

(0,1)

$4\pi 4 \sin \frac{1}{4}\pi x$

$$16\pi \sin\left(\tfrac{1}{4}\pi x\right) - \tfrac{2}{3}x^3 - 3x^2 + 4x \Big]_0^2$$

$$16\pi \sin\left(\tfrac{1}{4}\pi 2\right) - \tfrac{2}{3}(2)^3 - 3(2)^2 + 8 - 0$$

$$16\pi \sin\left(\tfrac{\pi}{2}\right) - \tfrac{16}{3} - 12 + 8$$

$$\boxed{R = 16\pi - \tfrac{16}{3} - 4 \text{ units}^2}$$

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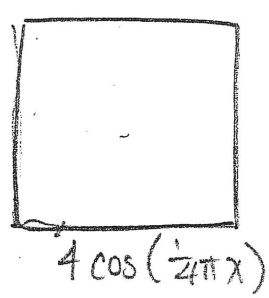
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right) - 4\right)^2 - \left(\left(2x^2 - 6x + 4\right) - 4\right)^2 \right) dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2 - \left(2x^2 - 6x + 4\right)^2 \right) dx$$



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Question 5

Overview

Students were given the graph of a region R bounded below by the graph of the function f and above by the graph of the function g, where $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. In part (a) students were asked to find the area of R, requiring an appropriate integral setup and evaluation. Students needed to correctly evaluate $\int_0^2 (g(x) - f(x))\,dx$. Part (b) asked for an integral expression for the volume of the solid obtained by rotating the region R about the horizontal line $y = 4$. Students needed to set up an integral where the integrand represents a cross-sectional area of a circular disc with inner radius $(4 - g(x))$ and outer radius $(4 - f(x))$. This yielded the integral $\pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$. Part (c) asked for an integral expression for the volume of the solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. Here the required integrand was $(g(x) - f(x))^2$.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student earned the integrand point and polynomial antiderivative point. The answer point was not earned due to incorrect distribution of the negative sign. In part (b) the student earned both integrand points. The student uses incorrect limits and did not earn the third point. In part (c) the student's work is correct.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (b). In part (a) the student earned the integrand point and polynomial antiderivative point. The answer point was not earned because of the incorrect distribution of the negative sign. In part (b) the student earned the first integrand point for one correct radius. Because the difference of the squares of the radii is reversed, the student did not earn the second integrand point. The constant of π is missing, so the third point was not earned. In part (c) the student did not earn the integrand point and therefore did not earn the limits and constant point.

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Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(1,\, 0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right)dx$

$$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right)dx$$

$$-e^{-y} = x^3 - 3x^2 + C$$

$$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$$

$$-e^{-y} = x^3 - 3x^2 + 1$$

$$e^{-y} = -x^3 + 3x^2 - 1$$

$$-y = \ln\left(-x^3 + 3x^2 - 1\right)$$

$$y = -\ln\left(-x^3 + 3x^2 - 1\right)$$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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6A₁

6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$$\frac{dy}{dx} = \cancel{e^0}(3-6)$$

$$\frac{dy}{dx} = -3$$

$$y - 0 = -3(x-1)$$

$$\boxed{y = -3x + 3}$$

$$y(1.2) = -3(1.2) + 3$$

$$\boxed{y = -.6}$$

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6A2

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$\int \frac{1}{e^y}\,dy = \int 3x^2 - 6x\,dx$$

e^{-y}

$$-e^{-y} = x^3 - 3x^2 + C$$

$$e^{-y} = -x^3 + 3x^2 + C$$

$$-y = \ln(-x^3 + 3x^2 + C)$$

$$y = -\ln(-x^3 + 3x^2 + C)$$

$$0 = -\ln(-(1)^3 + 3 + C)$$

$$0 = -\ln(-1 + 3 + C)$$

$$0 = -\ln(2 + C)$$

$$2 + C = 1$$

$$C = -1$$

$$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$$

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6B,

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$mT = e^y(3x^2 - 6x)$, $(1,0)$

$= e^0(3(1)^2 - 6(1))$

$= -3$

$y - y_1 = mT(x - x_1)$

$y - 0 = -3(x - 3)$

$y = -3x + 9$

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6B2

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$dy = e^y(3x^2-6x)dx$$

$$\frac{dy}{e^y} = (3x^2-6x)dx$$

$$\int e^{-y}dy = \int (3x^2-6x)dx$$

$$-e^{-y} = x^3-3x^2+C,\ (1,0)$$

$$-e^{-0} = (1)^3-3(1)^2+C$$

$$1 = 1-3+C$$

$$C = 3$$

$$-e^{-y} = x^3-3x^2+3$$

$$e^{-y} = -x^3+3x^2-3$$

$$-y = \ln(-x^3+3x-3)$$

$$y = -\ln(-x^3+3x-3)$$

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6C1

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6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$\frac{dy}{dx} = e^0(3(1)^2 - 6(1))$

$1(3-6)$

$y - 0 = -3(x-1)$

$\boxed{y = -3x+3}$

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6C2

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = e^y(3x^2-6x)\,dx$$

$$\frac{dy}{e^y} = \frac{\cancel{e^y}(3x^2-6x)\,dx}{\cancel{e^y}}$$

$$\int \frac{1}{e^y}\,dy = \int 3x^2-6x\,dx$$

$$\frac{2x^3}{3} - 3x^2$$

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Question 6

Overview

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation passing through a given point. Part (a) asked students to write an equation for the line tangent to the graph of f at the given point, and then to use this tangent line to approximate $f(x)$ at a nearby value of x. Students needed to recognize that the slope of the tangent line is the value of the derivative, given in the differential equation, at the given point. Part (b) asked for the particular solution to the differential equation that passes through the given point. Students should have used the method of separation of variables to solve the differential equation.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 1 point in part (a) and 5 points in part (b). In part (a) the student correctly computes the slope using the given derivative and earned the first point. The student gives a line through $(3, 0)$ rather than $(1, 0)$. The second point was not earned, and the student was not eligible for the third point. In part (b) the student correctly separates the differential equation and earned the first point. The student correctly antidifferentiates both the exponential function and the polynomial function, so the second and third points were earned. The student includes the constant of integration and earned the fourth point. The student uses the initial condition $(1, 0)$ and earned the fifth point. The last point was not earned because the student makes an arithmetic error while computing C.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (b). In part (a) the student correctly computes the slope using the given derivative and earned the first point. The student gives a correct tangent line and earned the second point. The student does not use the tangent line to approximate $f(1.2)$, so the third point was not earned. In part (b) the student correctly separates the differential equation and earned the first point. The student does not antidifferentiate the exponential or the polynomial correctly. The second and third points were not earned. The student did not earn any additional points in this part.

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